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Mesoscopic rectification in a quantum dot with spin–orbit interaction

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Abstract
We investigate the conductance of an open quantum dot in which uniform Rashba spin–orbit interaction (SOI) is present in the cavity region. The dot has a central triangular stopper (CTS) whose rotation angle controls the symmetry of the whole system. For a Fermi wavelength comparable to the linear dimension of the CTS, the SOI-dependence of the conductance is sensitive to both the direction of bias and the rotation angle of the CTS. We propose a quantum ratchet which generates the directed current against AC bias with time-average zero by using spin-polarized electron injection. The relationship between the symmetry of the dot and the rectification effect is revealed, and is used as a mechanism for charge rectification.

(Some figures in this article are in colour only in the electronic version)

Microscopic machines that rectify random motions into one expected direction have received a continued interest for a long time. Such machines, called ratchets, which are put in a non-equilibrium state under external forces with time-average zero, generate directed transport from random particle flows. The ratchet needs some spatial asymmetry of the system. A system proposed by Reimann *et al* [1] is noteworthy: particles in a one-dimensional asymmetric saw-tooth potential are periodically tilted by an external force with time-average zero under dissipation and thermal excitations (random forces), leading to a non-vanishing net current. But the model was classical–mechanical and macroscopic. In the context of nanodevices, Linke *et al* reported experiments on a single quantum dot that produces a ratchet effect for spin-unpolarized current [2, 3], which however had recourse to the deformation of electrostatic potential in the dot synchronized with the direction of bias voltage. Similarly, the breakdown of the Onsager relation was reported in the nonlinear response region of mesoscopic systems in the presence of a magnetic (B) field [4–6]. These reports also addressed the asymmetric deformation of electrostatic potential in the dot against the change of the sign of the B field. However, most of these works ignored the role of the spin degree of freedom (SDF). Is it possible to conceive the ratchet effect in the linear response region by incorporating the SDF?

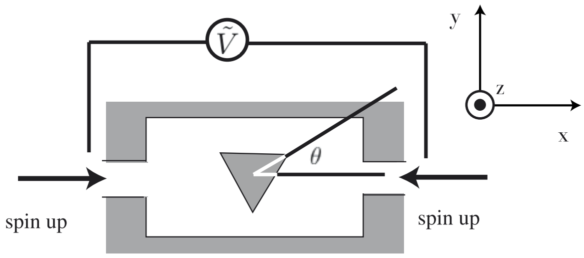

Figure 1. A schematic diagram of the dot with CTS. The z-direction is perpendicular to xy-plane. SOI distributes uniformly in the cavity region. Spin-polarized (spin-up in the z-direction) currents are injected from both sides.

On the other hand, from a viewpoint of spintronics [8–10], Rashba spin–orbit interaction (SOI) [7] has come to receive a growing attention, and is said to play the role of a spin-filter that produces spin-polarized current against the unpolarized injection of electrons [11–16]. The spatial distribution of SOI can be controlled by an external electric field applied to the dot [17, 18]. Interestingly, the SDF plays an essential role here despite the complete absence of the B field: Rashba SOI has nothing to do with the B field that is responsible for the Zeeman interaction. In this communication we propose a charge rectification of the fully spin polarized current, based on the quantum dot, which includes both a central triangular stopper (CTS) and uniform SOI in the cavity region.

A schematic diagram of the two-dimensional quantum dot that we propose is shown in figure 1. The rectangular dot has a rotational CTS with rotation angle denoted by θ. Uniform SOI is assumed to be present in the conductive region of the dot and absent in the leads. Throughout the text we assume: (1) the Fermi wavelength (λ) is comparable to the linear dimension (l) of the CTS; (2) an AC bias consisting of alternating ($\pm E$) plateaus is applied against the dot and spin-polarized currents are injected from both sides.

We use the recursive Green's function method [19, 20] to calculate the conductance. The conductance G from the left to the right leads is given by

$$G_{L\to R} = \frac{2e^2}{\hbar}\sum_{k,k'}\sum_{m,m'}|\langle k,+;m|\hat{T}|k',+;m'\rangle|^2, \tag{1}$$

where k, k' and m, m' denote modes and spin up (+1) or down (−1) in the lead regions, respectively. The second entry (+) in both initial and final states denotes the direction of electron propagation (from left to right). Similarly, $\langle k,-;m|\hat{T}|k',-;m'\rangle$ is the transmission amplitude from the right lead with mode k' and spin m' to the left one with mode k and spin m. Since we concentrate on the case of spin-polarized (z-polarization) injection, we suppress the summation over m' and fix to $m' = +1$. Then

$$G_{L(R)\to R(L)} = \frac{2e^2}{\hbar}T_{L(R)\to R(L)}, \tag{2}$$

where

$$T_{L(R)\to R(L)} = \sum_{k,k'}\sum_{m}|\langle k,+(-);m|\hat{T}|k',+(-);m'=+1\rangle|^2. \tag{3}$$

$T_{L\to R}$ and $T_{R\to L}$ are transmission probabilities (TPs) from the left to right leads and vice versa, respectively. The TPs are determined by using the Hamiltonian in the dot,

$$H = H_0 + V'(x,y), \tag{4}$$

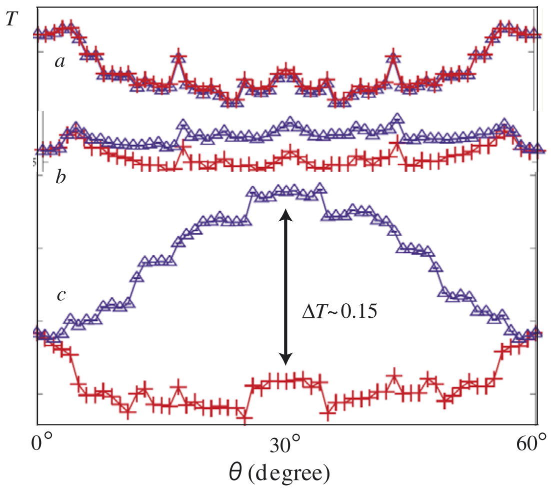

Figure 2. θ-dependences of $T_{L\to R}$ (red cross $+$) and $T_{R\to L}$ (blue triangle $\triangle$). $E_{\rm F} = 9$ meV: (a) $\alpha' = 0.5$, (b) 1.0 and (c) 2.0.

$$H_0 = \frac{\mathbf{p}^2}{2m^*}, \tag{5}$$

$$V'(x, y) = \frac{\alpha}{\hbar}(p_y\sigma_x - p_x\sigma_y) + V(x, y), \tag{6}$$

where $\alpha(\sim 10^{-11})$ eV m is the coupling constant of Rashba SOI and σ_x, σ_y and σ_z are Pauli matrices. $V(x, y)$ denotes the hard-walled confining potential due to the rectangular dot with CTS. For the lead width w, we choose the horizontal and vertical lengths of the rectangular dot and the length of a side of CTS to be $3w$, $1.6w$ and w, respectively. Further, let us define the dimensionless coupling constant α' as

$$\alpha' = \frac{\alpha m^* w}{\hbar^2} \tag{7}$$

with m^* the effective mass. α' will be varied in the range $0 \leqslant |\alpha'| \leqslant 3.5$, which corresponds to experimentally realistic values [8, 17]. The angle (θ) dependence of $T_{L\to R}$ and $T_{R\to L}$ for various values of α' is shown in figure 2. In our numerical calculation, we choose m^*, Fermi energy ($E_{\rm F}$) and w to be 6.1×10^{-29} g ($= 0.067 \times$ electron mass), 9 meV $< E_{\rm F} <$ 27 meV and 50 nm, respectively. In this case, the total mode number is two. Even for spin-polarized electron injection, it is obvious that $T_{L\to R} = T_{R\to L}$ at any angle θ, if $\alpha' = 0$. Surprisingly, however, we can see that $T_{L\to R}$ is different from $T_{R\to L}$ for nonzero α' in general, except at $\theta = 60° \times n (n = 0, 1, 2, \ldots)$, where the dot has up–down (UD) symmetry.

Noting the difference between $T_{L\to R}$ and $T_{R\to L}$, we proceed to apply an AC bias whose period is much longer than the timescale of the relaxation to a stationary state. We quantify the rectification effect ΔT with the use of TPs in stationary states as

$$\Delta T = (T_{L\to R} - T_{R\to L}). \tag{8}$$

The SOI-dependence of the rectification effect calculated for various angles θ are shown in figure 3. The dots with $\theta = 0°$ and 30° have up–down (UD) symmetry and left–right (LR) symmetry, respectively. The dots with $\theta = 10°$ and 20° have no such geometric symmetry. ΔT is not a monotonic function of α', and oscillates aperiodically with respect to α'. Figures 2 and 3 also reveal that no ratchet effect can be seen for the dot with UD-symmetry, and that UD-symmetry should be broken to see the ratchet effect.

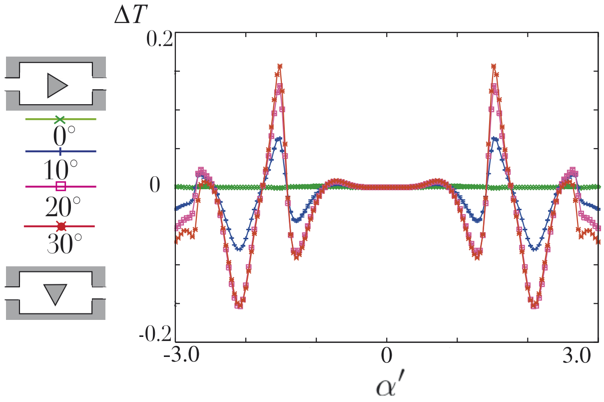

Figure 3. α'-dependence of rectification effect ΔT for $\theta = 0°$ (UD-symmetry), $\theta = 10°$, $20°$ and $30°$ (LR-symmetry). $E_F = 9$ meV.

Below, the theoretical analysis of the relationship between geometric symmetry of the dot and ratchet effect ΔT will be made in the case of two distinct (UD and LR) symmetries. We define the operator M_y, i.e., mirror inversion with respect to the xz-plane in both coordinates and spin spaces, which has the following properties.

$$M_y^{-1}\hat{y}M_y = -\hat{y}, \tag{9}$$

$$M_y^{-1}\hat{p}_y M_y = -\hat{p}_y, \tag{10}$$

and

$$M_y^{-1}\sigma_x M_y = -\sigma_x, \tag{11}$$

$$M_y^{-1}\sigma_y M_y = \sigma_y, \tag{12}$$

$$M_y^{-1}\sigma_z M_y = -\sigma_z. \tag{13}$$

Note: M_y in spin space with spin $\frac{1}{2}$ stands for the 2×2 unitary transformation U that satisfies $U^\dagger \sigma_i U = \sum_{j=1}^{3}(-1)^j \delta_{ij}\sigma_j$ with $i = 1, 2$ and 3 for x, y and z, respectively. Furthermore, noting that the leads themselves have UD-symmetry, we have

$$M_y|k, \pm; m\rangle = |k, \pm; -m\rangle, \tag{14}$$

where we have used equation (13). From analogy to the time-independent scattering theory, we use the Lippmann–Schwinger equation [21]

$$|\Psi\rangle = |\phi\rangle + \frac{1}{E - H_0 + \mathrm{i}\epsilon}V'|\Psi\rangle, \tag{15}$$

where $|\phi\rangle$ and $|\Psi\rangle$ are injected the standing wave and the scattered one, respectively. H_0 and V' are given in equations (5) and (6). Defining the transmission operator $\hat{T}$ as

$$\hat{T}|\phi\rangle \equiv V'|\Psi\rangle, \tag{16}$$

we can obtain an iterative solution for $\hat{T}$:

$$\hat{T} = V' + V'\frac{1}{E - H_0 + \mathrm{i}\epsilon}V' + V'\frac{1}{E - H_0 + \mathrm{i}\epsilon}V'\frac{1}{E - H_0 + \mathrm{i}\epsilon}V' + \cdots. \tag{17}$$

From the Hamiltonian in equation (6) and the properties of M_y in equations (9)–(13), we see

$$M_y^{-1}H_0 M_y = H_0. \tag{18}$$

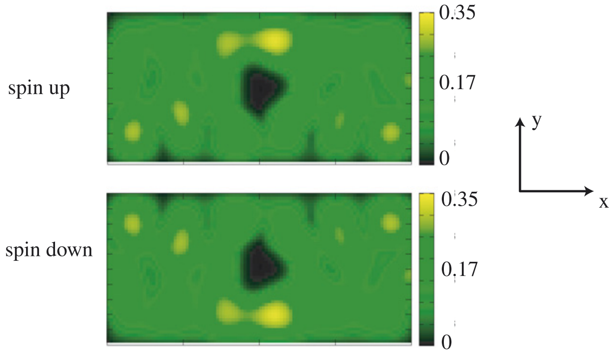

Figure 4. Probability density of wave function against the left injection of electrons with spin up (upper panel) and down (lower panel) in the dot with UD-symmetry. The case of mode 1 and $\alpha' = 2.0$.

In the case that the dot has UD-symmetry,

$$M_y^{-1} V' M_y = V', \tag{19}$$

because

$$M_y^{-1} V M_y = V. \tag{20}$$

Using equations (18) and (19), equation (17) leads to

$$M_y^{-1} \hat{T} M_y = \hat{T}, \tag{21}$$

and hence

$$\langle k, +; m | M_y^{-1} \hat{T} M_y | k', +; m' \rangle = \langle k, +; m | \hat{T} | k', +; m' \rangle. \tag{22}$$

Noting that the left-hand side of equation (22) is also equal to $\langle k, +; -m | \hat{T} | k', +; -m' \rangle$, with use of equation (14), we find

$$|\langle k, +; -m | \hat{T} | k', +; -m' \rangle| = |\langle k, +; m | \hat{T} | k', +; m' \rangle|. \tag{23}$$

Equation (23) together with equation (3) insists that TPs for spin up and down injection are identical if dots have UD-symmetry: $T^{\mathrm{up}}_{L\to R} = T^{\mathrm{down}}_{L\to R}$, where we let $T^{\mathrm{up}}_{L\to R}$ ($T^{\mathrm{down}}_{L\to R}$) be TP for an electron injected at the left lead with spin up (down). Analogously one can prove $T^{\mathrm{up}}_{R\to L} = T^{\mathrm{down}}_{R\to L}$. These equalities are consistent with the numerical evidence that the wavefunctions with spin-up injection is the mirror inversion (with respect to x-axis) of the one with spin-down injection in the case of UD-symmetry (see figure 4). Furthermore, the unitarity of the S-matrix demands that the total TP from the left to right leads is equal to the one from the right to left leads. Namely, $T^{\mathrm{up}}_{L\to R} + T^{\mathrm{down}}_{L\to R} = T^{\mathrm{up}}_{R\to L} + T^{\mathrm{down}}_{R\to L}$. Combining this fact with the notion below equation (23), it turns out that we get $T^{\mathrm{up}}_{L\to R} = T^{\mathrm{up}}_{R\to L}$ and $T^{\mathrm{down}}_{L\to R} = T^{\mathrm{down}}_{R\to L}$, that is, the same conductances (no ratchet effect), irrespective of the direction of spin-polarized (z-direction) current injected to the UD-symmetric dot.

In contrast, when the dot has LR-symmetry ($\theta = 30°$),

$$|\langle k, +; m | \hat{T} | k', +; m' \rangle| = |\langle k, -; -m | \hat{T} | k', -; -m' \rangle|. \tag{24}$$

This result can be obtained by using another inversion operator with respect to the yz-plane, M_x, in the similar way as shown for M_y applied to UD-symmetric dots, and verifies that $T^{\mathrm{up}}_{L\to R} = T^{\mathrm{down}}_{R\to L}$ and $T^{\mathrm{down}}_{L\to R} = T^{\mathrm{up}}_{R\to L}$. This partial symmetry in transmission does not suppress

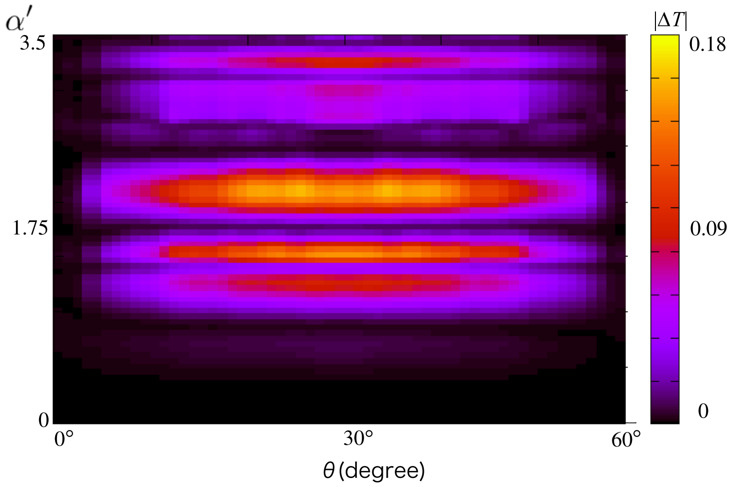

Figure 5. θ and α' dependence of $|\Delta T|$. Bright areas represent the enhanced rectification effect.

the rectification effect in our case where only spin-up injections are considered, and the TP for an electron injected with spin up at the left lead to be transmitted to the right one is not generally equal to the one from the right to left.

Finally we consider only the spin symmetry. Defining the operator (SM_z) acting only on the spin space as $SM_z^{-1}\sigma_x SM_z = -\sigma_x$, $SM_z^{-1}\sigma_y SM_z = -\sigma_y$ and $SM_z^{-1}\sigma_z SM_z = \sigma_z$, we obtain $SM_z^{-1}\hat{T}(\alpha)SM_z = \hat{T}(-\alpha)$. Hence, $|\langle k, \pm; m|\hat{T}(\alpha)|k', \pm; m'\rangle| = |\langle k, \pm; m|\hat{T}(-\alpha)|k', \pm; m'\rangle|$, which shows that the TP does not depend on the sign of the coupling constant of SOI, and explains why ΔT is independent of the sign of α' (see figure 3).

The dependence of $|\Delta T|$ on both rotation angle (θ) and SOI (α') is shown in figure 5, where $0^\circ \leqslant \theta \leqslant 60^\circ$ and $0 \leqslant \alpha' \leqslant 3.5$. For any value α', ΔT is vanishing for $\theta = 60^\circ \times n$ $(n = 0, 1, 2, \ldots)$, corresponding to the UD-symmetric dot, as predicted above. We can see that the largest rectification effect is obtained for the dots with $\alpha' \simeq 1.6$ and 2.1 for wide angles θ around $30^\circ + n \times 60^\circ$.

The nonvanishing SOI shifts the Fermi energy $\hbar^2 k_F^2/2m^*$ by $-\hbar^2 k_\alpha^2/2m^*$ to keep the electron density constant, where $k_\alpha \equiv m^*\alpha/\hbar^2$. Therefore it is essential to see the stability of the rectification effect against Fermi energy E_F. In figure 6 we investigate ΔT as a function of E_F and α' in the range $0 < \alpha' < 3.5$ and 9 meV $< E_F <$ 27 meV. The colour difference (greenish and reddish) denotes the direction of rectification. We find that the rectification effect is guaranteed in a wide range of E_F under an appropriately fixed value α', and is stable at finite temperatures up to 10 K.

The spin-polarized current is available by using an external spin filter based on a three-terminal dot which also employs SOI [15, 16]. Therefore neither the magnetization bath nor ferromagnets is needed. Our device will work as a quantum ratchet when bridged by a pair of such spin filters that provide the spin-polarized current. The dissipation should occur during the external spin filtering, which guarantees the second law of thermodynamics. So we can expect ballistic transport in the dot region without phase decoherence or Joule heating. This enables us to apply the Landauer formula or linear response theory. It should be noted that the asymmetry of transmission probabilities in the two-terminal conduction is traced back to the injection of the spin-polarized current and is compatible with the Onsager relation satisfied by the spin-unpolarized injection.

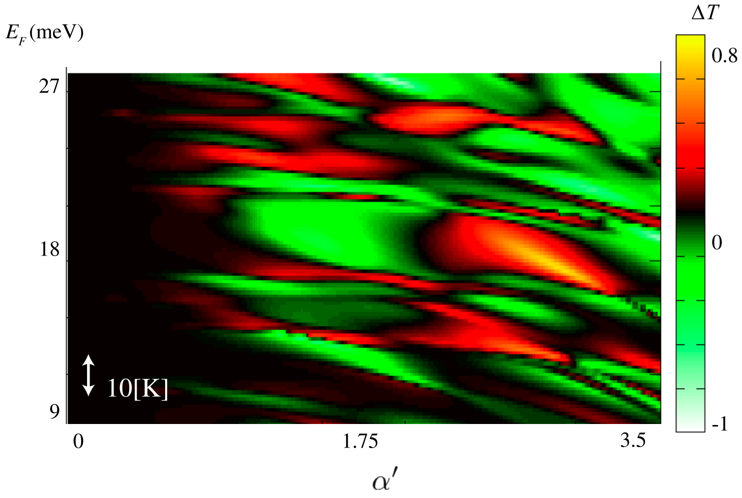

Figure 6. α' and E_F dependence of ΔT. Bright areas represent the enhanced rectification effect.

The advantage of this ratchet is that we can control the efficiency and direction of rectification (ΔT) by tuning the electric field responsible to the spin–orbit interaction. Further, this rectifier does not need any change of the self-consistent potential synchronizing with the direction of bias.

Although we have concentrated on z-spin-polarization, the present theory can be generalized to the case of other spin polarizations. An analogous symmetry argument requires the following conditions to see the rectification effect:

(1) Both UD-symmetry and LR-symmetry should be broken for x-polarized injection.
(2) Any symmetry is admissible for y-polarized injection.

In real experiments, completely spin-polarized injection is not easily available. The rectification effect is maximal for fully polarized injection and zero for unpolarized injection. The degree of rectification will be proportional to the polarization of the injection.

In conclusion, we have proposed a mesoscopic ratchet with the use of spin-polarized injection and uniform SOI. The relation between the geometric symmetry of dots and the rectification effect is revealed. It is shown numerically and analytically that an outstanding ratchet effect can be observed for a central triangular stopper (CTS) with rotation angle (θ) around $30° + 60° \times n$ ($n = 0, 1, 2, \ldots$). A rectification effect might also be available by changing the geometry of the exterior wall which breaks UD-symmetry, and for a linear array of such dots. The advantage of our ratchet is that it can be controlled by changing the coupling constant of SOI and the rotation angle of the CTS.

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